NIC # 15693 Network Analysis Corporation

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DATA OPTIONS FOR PACKET COMMUNICATIONS ON CATV SYSTEMS

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1. INTRODUCTION:

In Packet Radio Temporary Note #8, we showed that it was technically feasible to use existing CATV systems to transmit interactive packet data with very low error rates. In this report we discuss various system options to further enhance data transmission under different population densities and traffic constraints. We develop specifications on the hardware required to implement these options and show that they can be readily achieved and can be implemented with only minor modification to existing CATV systems.

We finally illustrate the use of the hardware and system options for the design of the Boston CATV system.

The detailed design for the Boston system first confirms our results in Packet Radio Temporary Note #8. Namely, for up to 3% active users the CATV system will handle all traffic at a data rate of 100 Kilobits/second for a slotted system. The new results indicated that an unslotted system will handle the same traffic with approximately one "frequency converter" per 20,000 terminals. To handle up to 15% active terminals approximately one multiplexer and three concentrators per 20,000 terminals are needed.

2. SYSTEM CONSTRAINTS:

In specifying hardware, a dominant consideration is the fact that the interactive packet cable transmission system must interface with a CATV system, a packet radio transmission system and an existing population of unsophisticated users. Thus, any hardware innovation must satisfy the following three classes of system constraints.

2.1 INTERFACE WITH CATV SYSTEM:

Two Way Options:

The data transmission system must be readily adaptable to a wide variety of existing CATV system designs and two way options.

Data Rates:

The data signals must not cause visible interference with video signals.

Installation:

If auxiliary data equipment is to be added to the CATV system, it must satisfy the following requirements:

A. It can be installed with only minor changes in the CATV System.

B. It need be installed in only a small number of locations.

C. It can be installed rapidly in early hours of the morning to prevent interference with TV service.

2.2 INTERFACE WITH PACKET RADIO SYSTEM:

Fixed Data Rates:

The equipment, such as modems, introduced at the terminal must use the packet radio rate of perhaps 100 kilobits/second.

Low Cost:

To maximize the marginal utility of data distribution over the CATV system, any equipment introduced must be inexpensive.

SYSTEM CONSTRAINTS:

If the packet radio system uses spread spectrum, the cable system can also be adapted to use spread spectrum. The modifications to the calculations in this report would be straight forward and the specifications to the equipment would change in data rates and bandwidth. The carrier frequency from the terminal onto the CATV system must be in the VHF range to use existing in-building CATV equipment. Hence, the packet radio system and the CATV system cannot use the same carrier frequency. Depending upon the data rates and bandwidths involved, the radio and CATV systems can be arranged to share some modules, at the baseband or IF frequency range.

2.3 INTERFACE WITH POPULATION:

Population Density Variations:

Standard transmission configuration options must be available for systems of various sizes, population densities and percent of active users. Because of the huge number of potential users, all terminal equipment must be simple and inexpensive.

Unsophisticated Users:

To minimize user interaction with the system operating mode, all terminal equipment must be the same for each location, it must use the same frequencies and data rates and it must have no options for equipment modification by the user.

2.4 CENTRAL TRANSMISSION AND LOCAL TRANSMISSION:

There are two modes in which the CATV data system may be used. In the "central" transmission mode all data passes through the head end for transmission to or from the longhaul ARPANET. The CATV system acts as a data gathering and data distribution system for the ARPANET.

In the "local" transmission mode, data may be directed between two terminals within the CATV system without use of the ARPANET. Depending upon the CATV system options available, this data may or may not be routed through the head end.

In our design, we must satisfy the requirements for both modes of operation.

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3. DATA SYSTEM OPTIONS:

We now consider the introduction of a number of basic data processing options into the CATV system in order to make our data transmission system adaptable to a variety of traffic requirements while satisfying the system constraints given in Section 2.

At present, it is anticipated that the CATV system will be used primarily in the central transmission mode. Furthermore, as we shall see, once the system specifications, parameters and hardware are determined for central transmission, only minor modifications are required to accomodate superimposed local traffic. For these reasons, we will emphasize the central transmission mode in our discussion.

We will use the terminology of the cable TV industry in describing the direction of signal flow. Signals traveling from the head end toward terminals will be said to be directed in the "forward" direction on a "forward" link and signals traveling from terminals toward the head end will be said to be directed in a "reverse" direction on a "reverse" link. A convenient synonym for "forward" will be "downstream", and for "reverse", will be "upstream".

3.1 CARRIER FREQUENCY CONVERSION:

In the simplest version of a data system, two carrier frequencies are used; one for forward transmission from the head end to the terminals, and one for reverse transmission from the terminals to head end. Let us call these angular frequencies ω_f and ω_r respectively. The next simplest option is to use frequency converters at a small selected set of points in the system. In the forward direction, the converter converts from ω'_f to ω_f and in the reverse direction, it converts from ω_r to ω'_r. The net result is that the terminals still receive and transmit at the frequencies ω_f and ω_r. However, in the trunk between the converters and the head end, there are four frequencies in use, ω_r, ω'_r, ω_f and ω'_f so that in these trunks twice the traffic can be handled.

The converter and the other devices to be described in this section are shown schematically in Figure 3.1.

The advantages of this scheme are:

a) All terminals are identical.

b) The converters also act as digital repeaters to reshape signals.

c) The capacity of the system is increased since two channels are available in each direction.

3.2 ROUTING:

In the interactive packet system, there is no requirement for routing since the basic premise is that all receivers listen to all messages that reach them and merely select the ones addressed to them. Nevertheless, we will consider the addition of some primitive low cost routing schemes to study their effect on system capacity.

In the central transmission mode there is no routing needed in the reverse direction since all messages reach the head end along the unique paths from the originating terminal. In the forward direction the signals at frequency ω'_f are blocked by filters at the converters and yields a simple form of routing.

In fact, at any section of trunk not requiring signals at ω'_f filters can be added to block ω'_f. Adding these filters does not increase system capacity but may be useful if the frequency ω'_f can be used for local signaling when not being used for data transmission.

At any junction containing a converter, digital routers must be added to send messages at ω_f down the trunk to which they are addressed rather than all trunks. Such a router may be added at any other point in the system as well. This is called "forward routing" and can increase system capacity. Forward routing requires a digital router which can read and interpret message addresses.

Let us now consider these system options in the presence of local traffic. The option of frequency conversion is uneffected and perform in exactly the same manner as in the central transmission mode. However, an extra routing option is available for local transmission. In particular, if two terminals are on the same trunk, then the message between them can be intercepted and routed at a routing station rather than travel all the way to the head end. Such routing is called "local routing". Local routing reduces the traffic on the main trunk.

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3.3 MULTIPLEXING:

As the next more complicated option, the data rate as well as the carrier frequency is changed at a converter,* i.e. a multiplexer can be used. The advantages of this arrangement are:

A. All terminals operate at low data rate compatible with the packet radio system.

B. On heavily used lines near the head end a higher data rate, say one megabit/second, can be used to increase the number of potential active users or decrease the delay.

C. Even though a section of the trunk can carry high data rate traffic at carrier frequencies ω'_r and ω'_f other users can still use the system at the low data rates at ω_r and ω_f. Thus the number of multiplexers required is small.

D. At the low data rate, signal distortion is kept to a minimum.

3.4 CONCENTRATION:

Finally, the multiplexer at junctions may be replaced by a concentrator. That is, messages arriving simultaneously on two or more links in the reverse direction are buffered and sent out sequentially at the higher data rate.* This essentially makes the system downstream from the concentrator appear to operate at the higher data rate and hence increases the system capacity even further.

With the use of converters, multiplexers, concentrators and routers, we have a highly flexible interactive packet data system which obviously meets most of the system requirements of capatibility with the CATV system, the packet radio system and the population. In particular, the system has the following characteristics:

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* The descriptions of the multiplexing and concentration options are adaptions of these terms to the interactive packet mode of operation and hence, they do not correspond exactly to the usual descriptions for standard FDM and TDM systems.

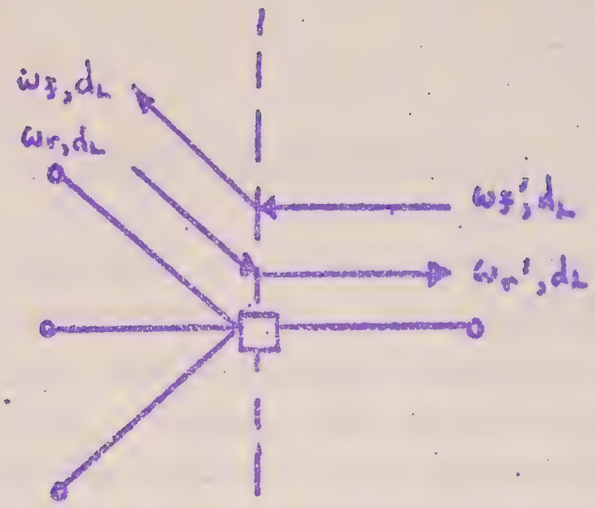
A. The number of active users that may have access to the system can be readily controlled by varying the number of converters, multiplexers, concentrators and routers.

B. The transmission rates at every terminal are at the low data rate of 100 kilobits/second, associated with the packet radio system.

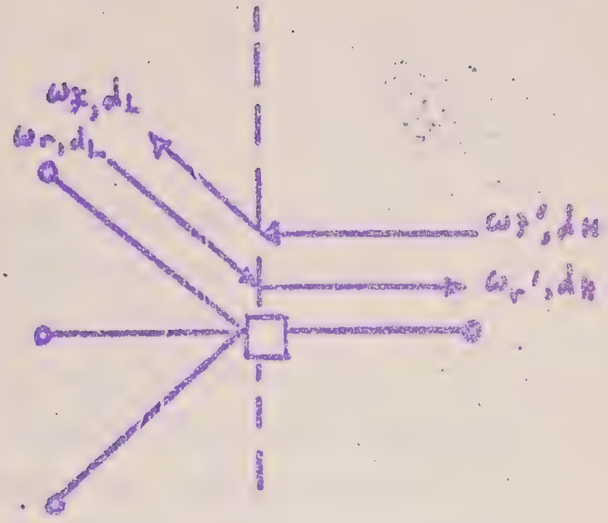
C. Terminal equipment is inexpensive because modulation takes place at the low data rate.

D. All terminals receive and transmit at the same frequencies and data rates.

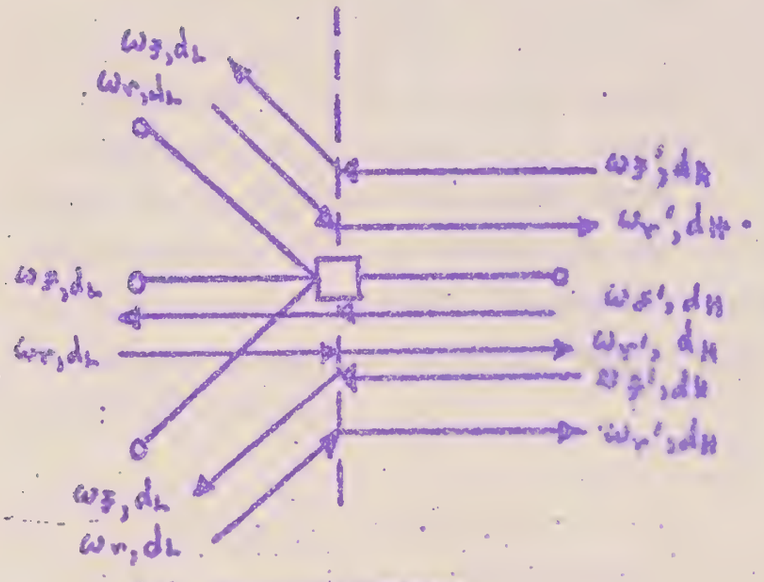
We shall show in subsequent sections that the introduction of these system options meets all of the remaining system constraints as well. In particular, we will consider each piece of hardware in detail and show that the specifications for the equipment can be met practically and that the equipment can be readily incorporated into existing CATV systems. Finally, we derive formulae for the traffic capacity of the system as a function of the demands and the equipment location. Using these formulae, we then consider typical designs for the Boston CATV system and show that only a very few additional pieces of equipment are required to accommodate a broad range or demands and requirements.



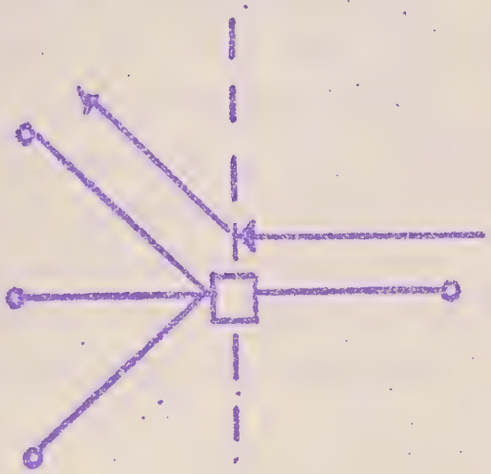
(a) converter



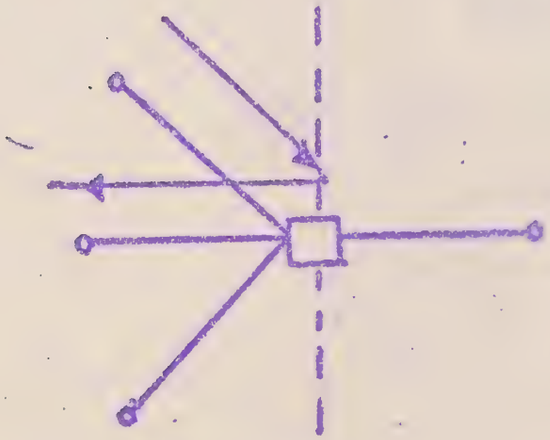
(b) multiplexer



(c) concentrator



(d) forward router



(e) local router

Figure 3.1 Device schematics; devices indicated by squares. Carrier frequencies ω_r, ω'_r, reverse; ω_f ω'_f, forward. High data rate, d_H; Low data rate, d_L.

4. HARDWARE SPECIFICATIONS:

Based upon over-all system specifications and options, we now develop functional descriptions and specifications for the required equipment. The specifications are not intended to represent the tightest requirements that can be met by advanced designs. Rather, the specifications can be comfortably met by existing CATV and data equipment and will meet the requirements for picture quality and bit error rates discussed in Packet Radio Report #8.

4.1 BAND SEPARATION FILTERS:

The most common two-way CATV configuration at present is the simplest single-trunk subchannel split in Figure 4.1. A much more sophisticated and flexible system is the dual trunk "feederbacker" arrangement in Figure 4.2, which is the system used in Boston. For both these systems we must have an inexpensive, simple method of interfacing with a converter or router to be added after the construction of the CATV system has been completed. Furthermore, the interface must be readily adaptable to other possible two-way configurations.

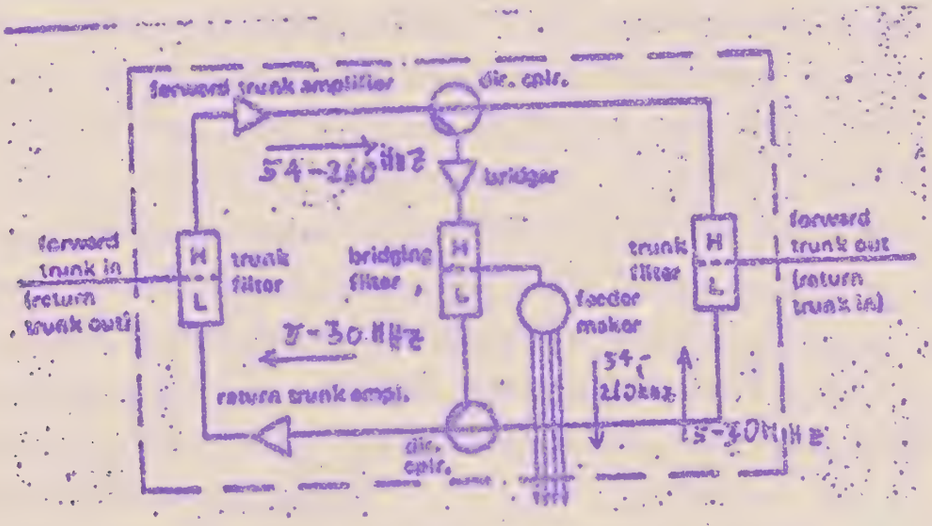


Figure 4.1 Simplest two-way configuration

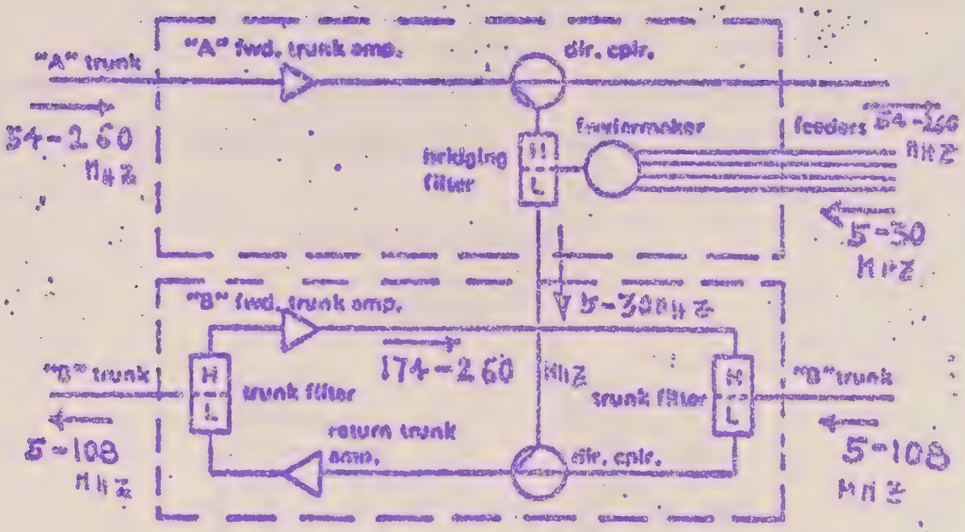


Figure 4.2 "Feederbacker" Configuration

The problem is readily solved by use of a pair of diplex filters as shown in Figure 4.3 or in some cases by a pair of triplex filters as shown in Figure 4.4. The units can be housed in casings a few inches in each dimension and are readily installed on line. The passbands of the filters can be set for the particular system transmission frequencies. The conditions for perfect band separation are that the passbands of the filter F_f, F_r and F_t do not overlap and that the passbands cover the full frequency range of the channel. Thus, for the two-way configuration in Figure 4.1, the triplex filter is used with the following passbands.

F_f passes the signals at carrier frequencies ω_f and ω'_f

F_r passes the signals at carrier frequencies ω_r and ω'_r

F_t passes all frequencies not passed by F_f and F_r.

F_f therefore isolates the forward channel for conversion or routing and F_r, the reverse channel.

For the feederbacker configuration in Figure 4.2, the diplex filter is used for line A with F_f and F_t specified as follows:

F_f passes the signals at carrier frequencies ω_f and ω'_f

F_t passes all frequencies not passed by F_f.

For line B, the diplex filter is used with the following passbands:

F_r passes the signals at carrier frequencies ω_r and ω'_r.

F_t passes all frequencies not passed by F_r.

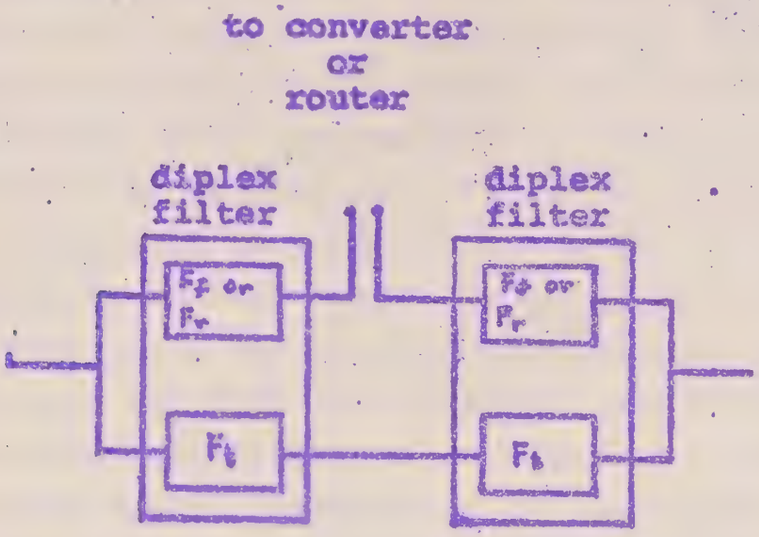


Figure 4.3 Diplex Band Separation Filter

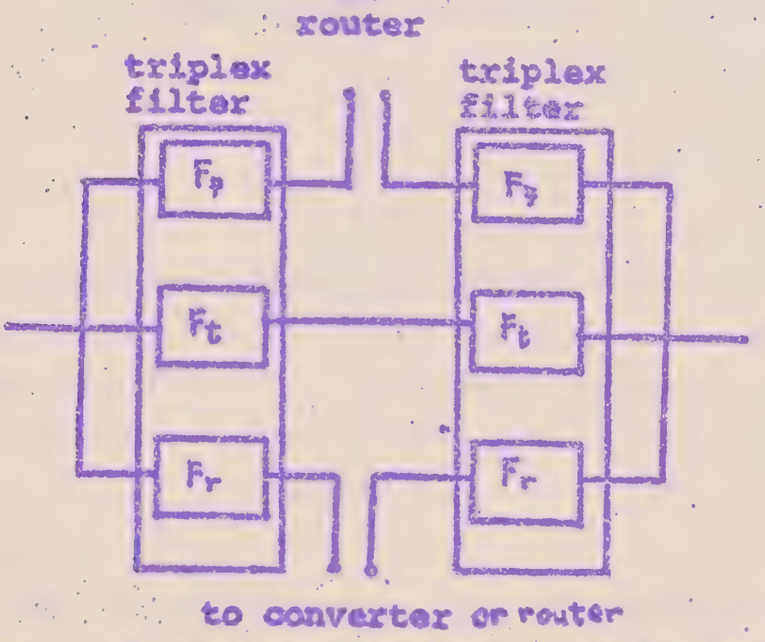
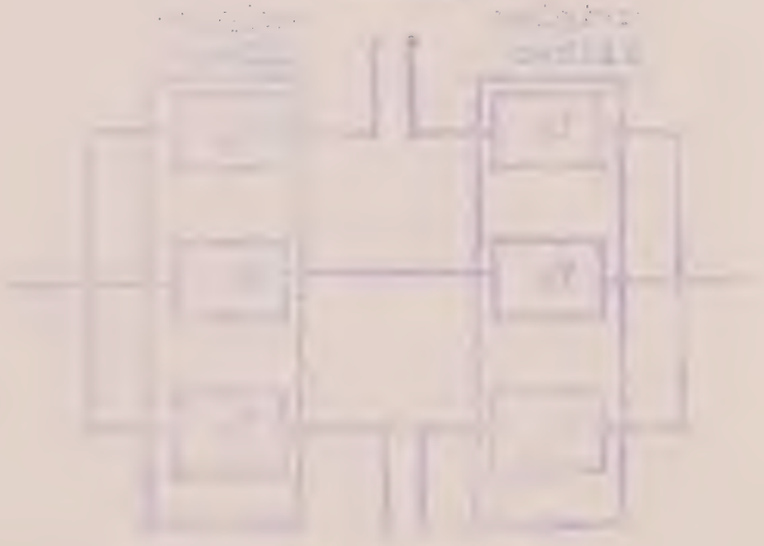


Figure 4.4 Triplex Band Separation Filter



For standard CATV circuits, each band separation unit adds a flat insertion loss of at most, 1.75 db. To make up this loss, the gain of the subsequent amplifier might be increased by 3.5 db. With most lines of equipment, this simply means reducing the value of the input attenuation pad by 3.5 db. However, if the attenuator is already set at too low a value, then the 3.5 db can be made up by increasing the size of the cable feeding into the band separation filter. For example, for a broad range of foam dielectric coaxial cables, the difference between the losses of .5 inch and .75 inch coaxial cable at channel 13 is about .4 db/100 ft. Thus, less than 900 feet of cable would have to be converted from .5 inch to .75 inch. This change would necessitate that at most ten subscriber taps be changed in value and that the equalizer for one amplifier be adjusted--all minor adjustments to an existing system.

The required specifications for the individual filters are consistent with typical parameters for CATV bandpass and notch filters. For example, the characteristics below are more than adequate for the data system and are derived from off-the-shelf CATV filters.

FILTER SPECIFICATIONS

Center Frequency	in the range 30-260 MHZ
Bandwidth	5.5 MHZ at .5 db points
Insertion Loss	1.25 at low VHF to 1.75 at high VHF
Impedance	75 Ω ,-23 db reflection coefficient
Passband Group Delay Variation	10 nanoseconds max
Passband Ripple	.01 db max
Stopband Attenuation	at least 30 db at .5 db bandedge plus 1 MHZ

Table 4.1

4.2 MODEMS:

For the sake of illustration, we will assume that differential quatrature phase shift keying is used as in the packet radio system.

Vertical

Using DQPSK we can certainly meet the error rate requirements discussed in Packet Radio Report #8.

MODEM SPECIFICATIONS

	MODULATOR	DEMODULATOR
type of modulation	DQPSK	DQPSK
data throughput	100 KB/sec.	100 KB/sec.
carrier frequency	5-30 MHZ	54-66 MHZ
nominal signal level	output 32 dbmv at 240 MHZ	input 10 dbmv at 240MHZ
impedance	75Ω	75Ω
reflection coefficient	-25 db	-23 db
noise figure	7.5 db	7.5 db
power hum	60 db below signal level	60 db below signal level

Table 4.2

Modems meeting these specifications are available with performance parameters and resulting system degradation indicated in Table 4.3 taken from [Cuccia, 1973]. The signal degradation is within acceptable limits for error rates discussed in Packet Radio Report #8.

MODULATOR

Carrier Instability in Carrier Source	1/2°RMS in a PLL with Bandwidth=0.03% Bit Rate Bandwidth	0.05 dB
Static Phase Error in QPSK Modulator	2°	0.10 dB
Rise Time in Each Phase Change	1/4 Bit Period	0.40 dB
Amplitude Unbalance in QPSK Modulator	0.2 dB, 10°/dB	0.10 dB
Data Asymmetry from Data Source	±2%	0.05 dB
Group Delay Distortion (Modulator)	25°	0.20 dB
Clock Instability	1°RMS in the Bit Synchronizer PLL	0.10 dB
	SOURCE TOTAL =	1.00 db

DEMODULATOR

Incidental FM from all Oscillators in RF Channel and Carrier Reconstruction	1/2°RMS in a PLL with Bandwidth=0.03% of Bit Rate Bandwidth	0.05 dB
Static Phase Error in QPSK Demodulator	2°	0.10 dB
Reference Phase Noise in Reference PLO	1°	0.10 dB
Total Group Depay Distortion (Total Channel from Modulator)	20°	0.35 dB
Timing Jitter in Matched Filter Sampler	4%	0.25 dB
Timing Bias Error in Matched Filter Sampler	1.5%	0.15 db
DC Offsets-Total Receive	4%	0.10 dB
Data Waveform/Matched Filter Detector Mismatch	—	0.60 db
	SOURCE TOTAL =	1.70 db

Table 4.3

4.3 CONVERTERS:

The converter specifications are in Table 4.4.

function	convert carrier frequencies in VHF range
input impedence	75Ω ,-23 db reflection coefficient
output impedence	75Ω ,-23 db reflection coefficient
oscillator accuracy	.005%
conversion gain	5 db min
noise figure	10 db in analog mode
amplitude variation	± .75 db over 6MHZ Bandwidth

Table 4.4

4.4 MULTIPLEXERS, CONCENTRATORS AND ROUTERS:

The multiplexers, concentrators and routers all operate digitally at the same frequencies and data rates and differ only in details and relative memory and logic requirements. The specifications for all three are given in Table 4.5.

	Multiplexer	Concentrator	Router
number of inputs	≤3	≤3	≤3
number of outputs	≤3	≤3	≤3
input data rate	100KB/sec	100KB/sec	100KB/sec or 1 Megabit/sec
output data rate	1 Megabit/sec	1 Megabit/sec	100KB/sec or 1 Megabit/sec
throughput (packets)	1 packet/470 m sec	1 packet/470 m sec	1 packet/470 μ sec for 1Megabit operation
packet length	460 bits/packet	460 bits/packet	460 bits/packet
carrier frequency	VHF range	VHF range	VHF range
core memory requirement	two packets	six packets	two packets

Table 4.5

In addition, we can assume timing errors similar to the errors for the modem, yielding a total degradation of 1 db for signal reception and 1.7 db for signal transmission.

5. LOCATION OF AUXILIARY DATA EQUIPMENT:

As mentioned previously, the band separation filters can be inserted anywhere in the CATV system with no difficulty and can then feed any converter, multiplexer, concentrator or router. If these units are less than about 3 cubic feet in dimension and 20 lbs. in weight, they are roughly the same sizes as standard existing CATV equipment and can be readily located on telephone poles or in conduit. If the equipment is too large to be physically located on line, then it can be located in specially designated stations. In many two-way systems "local origination stations" are scattered throughout the system and are in effect local TV studios. There is certainly room in them for additional equipment. Otherwise, any apartment can be made into a data station. In either case, cable must be installed from the band separation filter to the station and back. If need be, amplifiers can be added on these "drop" lines since they affect only the data transmission frequencies and not the TV channels.

A primary consideration in the location of converters, multiplexers, concentrators, and routers is the traffic capacity of the system in both the local and central modes.

In appendices A and B we therefore develop simple descriptive formulae for the number of messages in a link in the network as a function of the location of converters, multiplexers, concentrators and routers. We first consider the traffic as a function of routing. Once this is done, we will see that the converters, multiplexers, and concentrators are readily taken into account. Using these formulae, we can then try various designs for the Boston area.

6. BOSTON DESIGN:

To determine the usefulness of the various options and devices we have considered, we will apply them to the design of an interactive packet data system for Medford, Massachusetts, a section of the Boston CATV complex. In Figure 6.1, a branch of the trunk is drawn for Medford, Massachusetts. The triangles represent bridger amplifiers. These amplifiers feed into feeder cable and extender amplifiers with customer taps and drop lines emanating from the feeder cable. In the design for Medford, the feederbacker arrangement is used so that the trunk lines are dual cable and the amplifiers are two-way units as shown in Figure 4.2. The hexagons represent local origination stations.

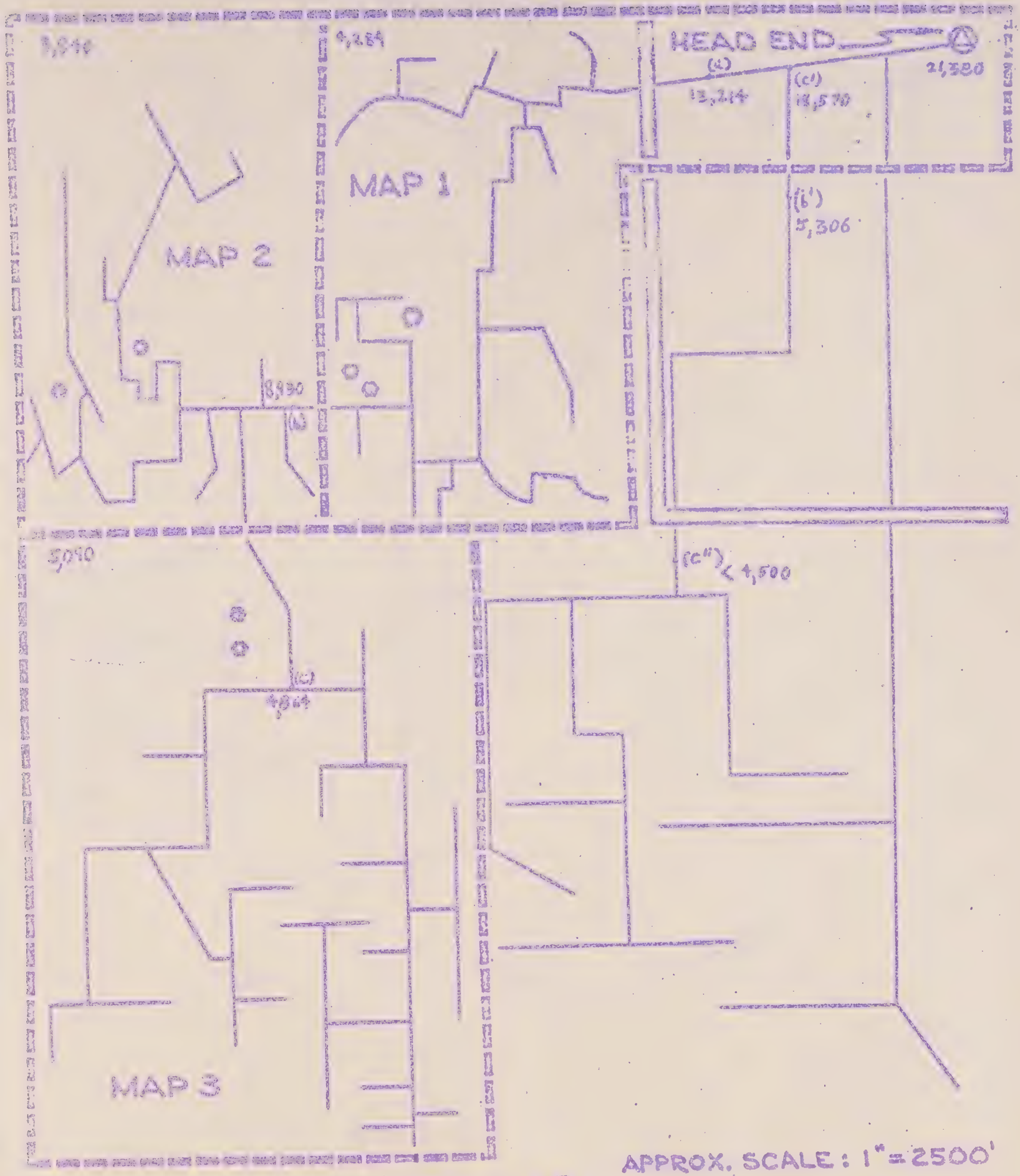
The feeder system emanating from a given bridger amplifier is called a cluster. The number next to each amplifier gives the number of terminals in the cluster associated with that amplifier. The average number of terminals per cluster is 137 with complete coverage of all homes.

We now consider the design of the Boston interactive packet cable system by focusing our attention on one trunk in the Medford area. As in Packet Radio Report #5, we assume that the average traffic per terminal is 40 bits/sec. We conservatively assume that this is the rate for inquiries as well as responses. We assume that in the design the data from the terminals is 100 kilobits/sec; the upconverted rate is 1 Megabit/sec. From Packet Radio Report #5, we obtain the results in Table 6.1 showing the number of active terminals that can be supported on a trunk at each data rate.

type of system / data rate	slotted system	unslotted system
1 Megabit/sec	9,000	4,500
100Kilobit/sec	900	450

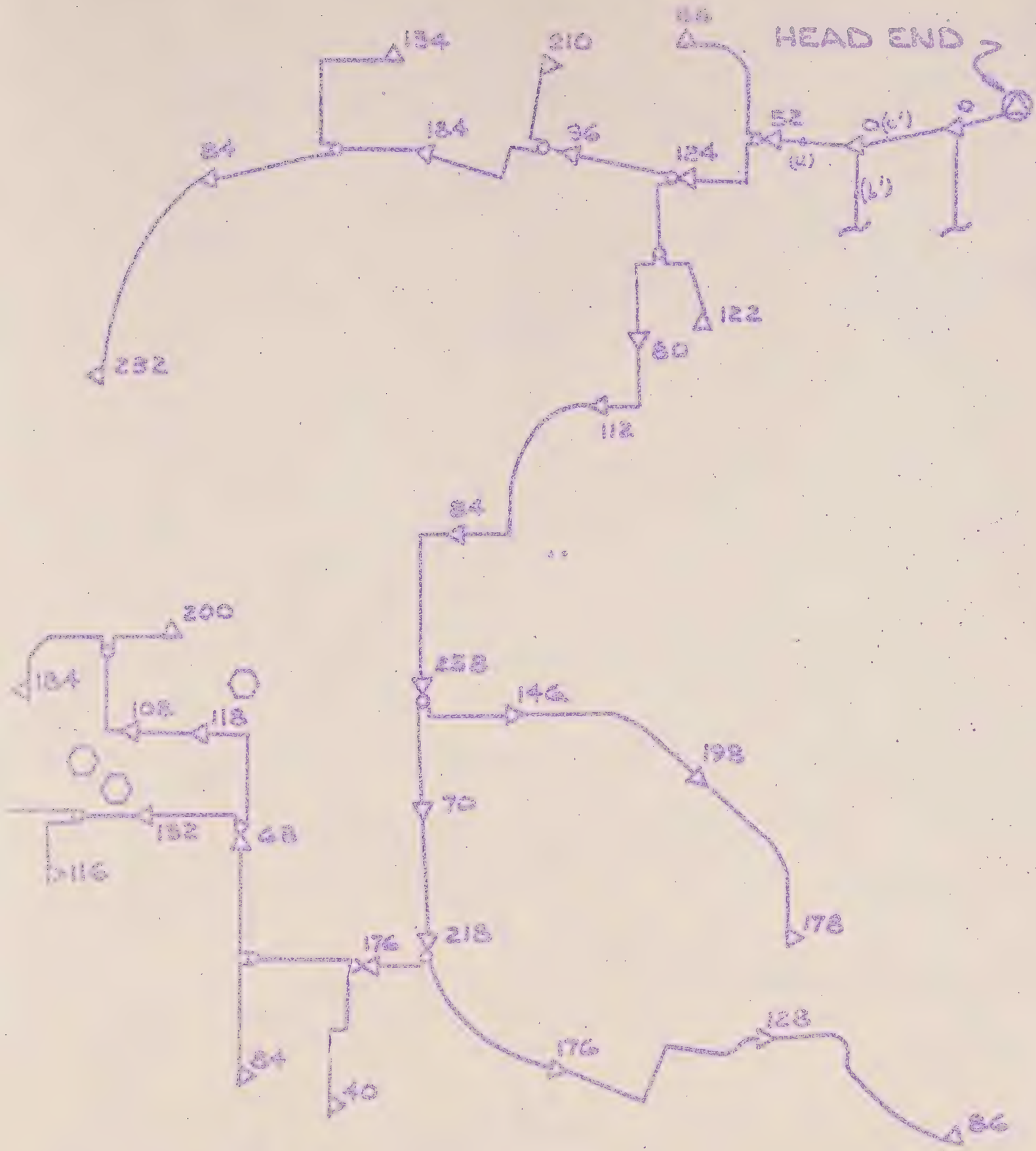
Table 6.1 Number of Active Users Per Trunk

MEDFORD MALDEN



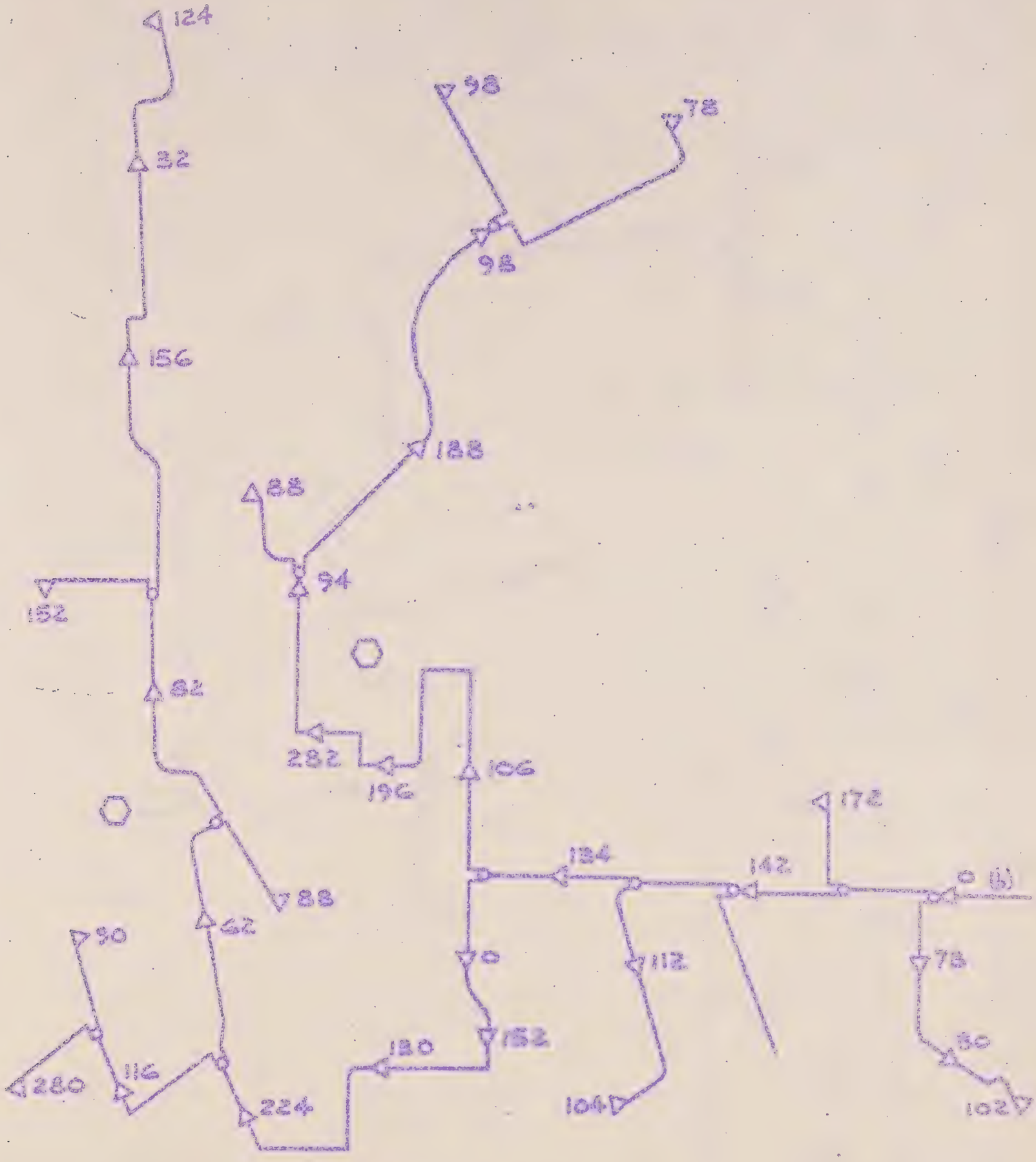
KEY MAP

Figure 6.1a



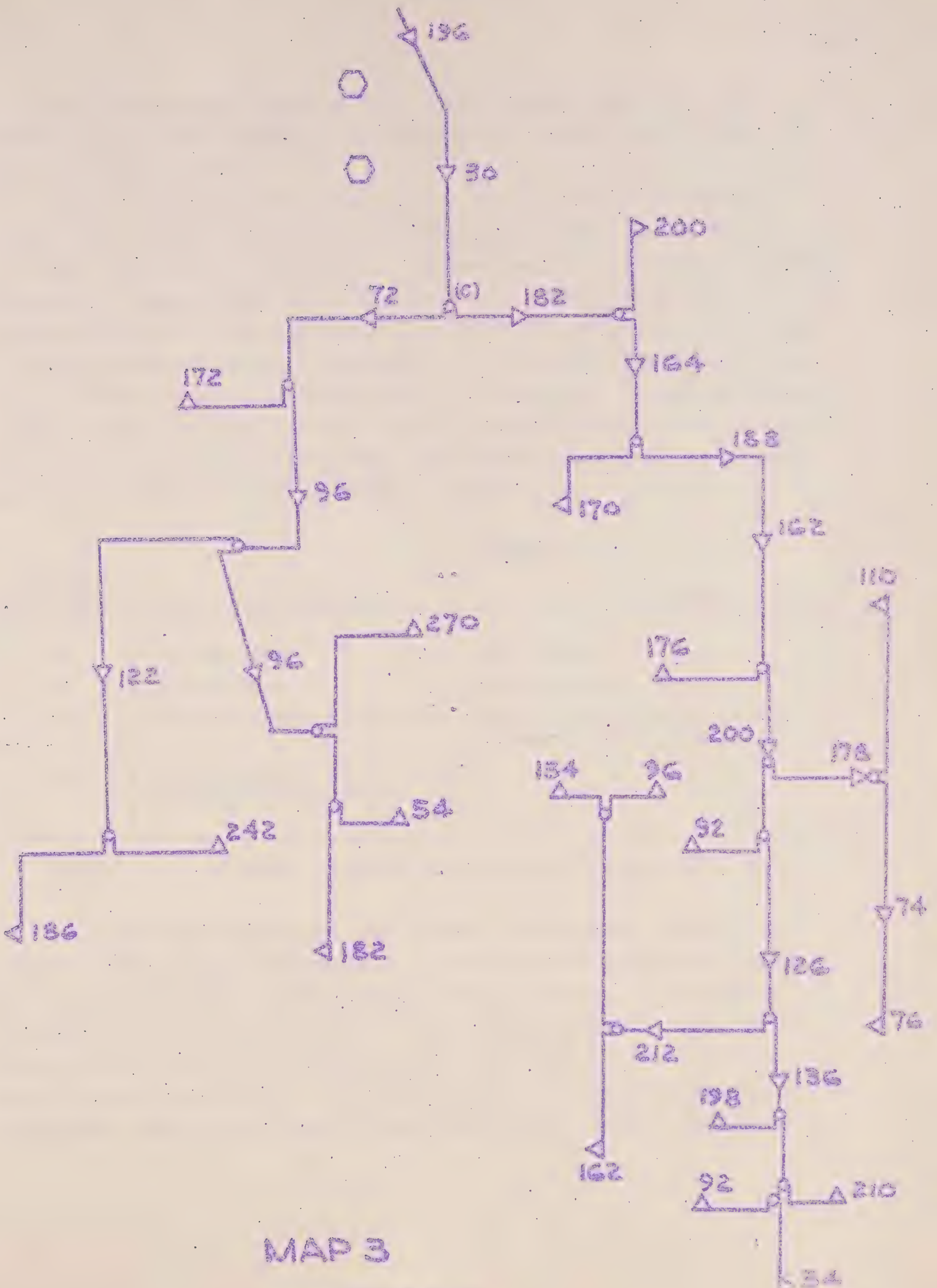
MAP 1

Figure 6.1b



MAP 2

Figure 6.1c



MAP 3

Figure 6.14

We first design the Medford trunk for the case in which there is no local traffic. We design the system for 1%, 3%, 10% and 15% active users.

Since we do not yet have a firm idea of relative costs of converters, concentrators, multiplexers and routers, we are in no sense optimizing the design. We are merely presenting feasible designs to demonstrate the wide range and flexibility achieved by combinations of a few devices. The designs are easily described by simply indicating the location of the various devices on the map in terms of an alphabetic label on the map. To aid in visualizing the design, the number in the rectangle beside the letter (on the key maps) indicates the population downstream from that point. The designs are as shown in Table 6.2.

% active terminals	DESIGN	
	Slotted	Unslotted
1%	no devices	no devices
3%	no devices	converter at (a)
10%	multiplexers at(b)&(b')	multiplexers at(c'') and concentrators at(b),(c), &(c')
15%	multiplexers at(c'') and concentrators at(b),(c) &(c')	

Table 6.2 Feasible Designs for Central Transmission Mode

Suppose now that X% of the traffic from every terminal is local traffic whose destination is uniformly distributed throughout the network and suppose there are no routers, converters, multiplexers or concentrators. Then every local message must go from the originating terminal to the head end on reverse links and from the head end to the destination terminal on reverse links. Therefore, each local inquiry traverses both forward and reverse

links instead of only reverse links as for central transmission. Similarly, each response occupies both forward and reverse links rather than only forward links as for central transmission. The net effect is that for X% local traffic, traffic in the branches is the same as if all the traffic were in the central transmission mode but the population were increased by X%. Hence, the previous design procedures are still applicable and the design already given can be used for local transmission superimposed on the central transmission provided the number of active users are appropriately adjusted. For example, the design for 10% active terminals in the central transmission mode can be used for 5% active terminals with 25% of the traffic on a local basis. In addition, as an example, if there were heavy traffic among the stations downstream of point (d), this might be handled by a local router at (d).

..

7. CONCLUSIONS:

In Packet Radio Temporary Note #8, we showed that it was technically feasible to use existing urban CATV systems to transmit interactive packet data under the following conditions:

A. All terminals operate at the packet radio data rate of say, 100 Kilobits/sec.

B. The average message rate of a terminal is 40 bits/sec.

C. Approximately 3% of the terminals are active at any time.

The next question which then arises is whether the CATV system can be practically augmented if the message rate or percentage of active users is significantly greater than the above numbers. This question has been answered in the affirmative in this report. We have introduced rather simple converters, concentrators, multiplexers and routers which meet the specifications of buildable equipment and enable us to increase the number of allowable active terminals by five-fold in the sample design of the Boston system, using only 3 concentrators and 1 multiplexer per 20,000 terminals. Furthermore, with the use of the diplex and triplex filters specified in the report, these units are readily added to an existing CATV system without interruption of video service.

The next steps in the study of interactive packet data transmission over CATV systems involve some problems of theory and some more interesting problems of implementation. In this report we presented formulae for the traffic in the net. Using these formulae we will next develop an algorithm for the optimal location of converters, multiplexers, concentrators and routers. Based on these locations, we can then make a detailed comparison of interactive packet transmission over cable versus other alternatives involving combinations of polling and frequency and time division multiplexing, as well as with an "over-the-air" packet radio system.

The more important part of the study involves the actual implementation of the system. We are, after all, dealing with

existing CATV system with parameters fixed by uncontrollable factors. Since we have formulated specifications for data equipment and system specifications at an early stage, we must begin a series of system tests and simulations to actually select modulation schemes and evaluate equipment performance. The following experiments should be performed sequentially:

A. Transmit data at 100 Kilobits/sec. over a video channel from a single source to determine error rates for data and effect on video signals.

B. Repeat A at data rates from 1 Megabit/sec. to 6 Megabits/sec.

C. Repeat A and B with interactive messages from multiple sources.

D. Repeat A - C with the introduction of converters, multiplexers, concentrators and routers.

E. Determine CATV data modem configuration for optimal interface with packet radio modem.

8. APPENDIX A: NOTATION:

In order to precisely formulate the traffic requirements throughout the network, it is convenient to introduce some elementary descriptive terminology from graph theory.

We define a set of points called nodes to represent the head end, junction points, router, converter, concentrator, multiplexer and bridger locations. The nodes are represented by integers; 0 for the head end and, 1 to n for the remaining nodes, where the total number of nodes is n + 1. The transmission system including amplifiers and any single or dual cable joining two points corresponding to nodes a and b, is represented by an undirected arcs[a,b]. The arcs and nodes together comprise a graph G = (N,A) where N is the set of nodes and A is the set of arcs. G is a tree, in our case, that is a graph joining all nodes and containing no "cycles".

Let $P_{i,j}$ be the "path" in G from node i to node j, that is the sequence of nodes and arcs listed in order in tracing a route from i to j. If node 0 is the head end, then path $P_{0,k}$ is a path from the head end to node k. If node i precedes node j in $P_{0,k}$ we say that node i is upstream of node j and node j is downstream of node i.

$$i = u\ (j)$$
$$j = d\ (i)$$

Let k=d(j), and i=d(k), or i=k then we write;

$$i = d(j,k)$$

$$D(j,k) = \{ i \mid i = d(j,k) \}$$

Let D(j) be the set of all downstream points of j.

$$D(j) = \{ i \mid j = d(i) \}$$

Let A x B be the cartesian product of the sets A and B, that is, $A \times B = \{(a,b) \mid a \in A,\ b \in B\}$

Let I(a,b) be the number of inquiries originating at node a addressed to node b.

Let R(a,b) be the number of responses originating at node a addressed to node b.

Then define:

$$I(A,B) = \sum_{(a,b) \in (A \times B)} I(a,b)$$

$$R(A,B) = \sum_{(a,b) \in (A \times B)} R(a,b)$$

$$M(A,B) = I(A,B) + R(A,B) \quad ..$$

EXAMPLE:

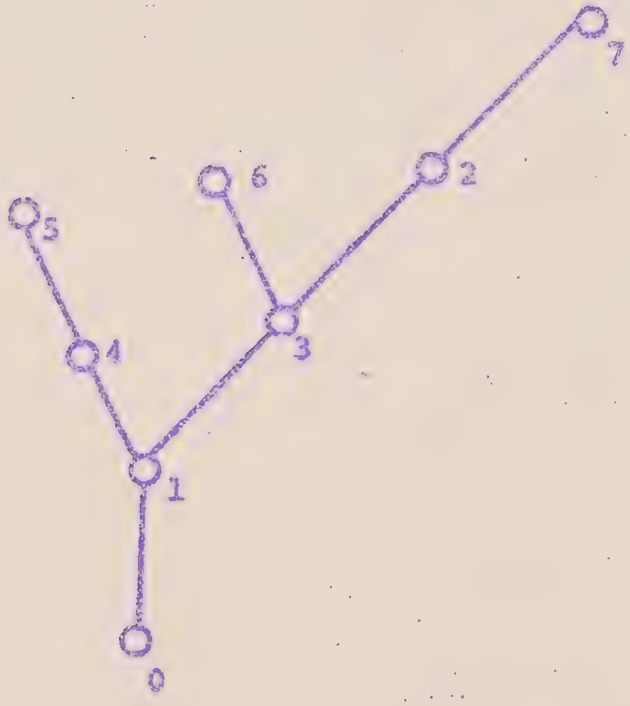


Figure 2.1

$N = \{0, 1, 2, 3, 4, 5, 6, 7\}$

$2 = u(3)$

$2 = u(1)$

$2 = u(1,3)$

$D(1,3) = \{2, 6, 7\}$

$D(1) = \{2, 3, 4, 5, 6, 7\}$

For $A = \{4,5\}$ and $B = \{2,3\}$

$$M(A,B) = I(4,2) + I(4,3) + I(5,2) + I(5,3)$$
$$+ R(4,2) + R(5,3) + R(5,2) + R(5,3)$$

9. APPENDIX B. FORMULAE FOR TRAFFIC IN LINKS:

In this section we present formulae for the link traffic in terms of the location of concentrators, multiplexers, converters and routers. We first assume there are only converters and routers. We use the notation developed in Appendix A with intuitive explanations of the significance of the notation.

Let us consider the arbitrary junction in the network and designate it by the integer i. We are interested in the traffic in links downstream from i.

There are three cases to consider:

A. There is local routing at i.

B. There is forward routing at i.

C. There is no routing at i.

The notation used is defined precisely in the terminology of graph theory in Appendix A. A simple intuitive explanation is given in this section.

We assign the integer 0 to represent the head end and we assign a unique integer to every junction, bridger*, multiplexer, concentrator and converter location in the system. Let N represent the integers corresponding to all these locations. We wish to consider the traffic in a specific link downstream from the integer i representing the junction under consideration. Let i' represent the integer corresponding to the first point downstream from i. We wish to know the traffic in the link between i and i', [i,i']. The traffic in [i,i'] depends upon the local routers downstream from i and the forward routers upstream from i. We therefore introduce some special terminology to represent these points. Trace any path from i in the downstream direction so the path contains i'. The first local router on this path (aside from i itself) is designated $t_{i,\ell}$ where ℓ is the number of the point at which the router is located. If there is no local router on the path, then the last point on the path is called $t_{i,\ell}$. Next, trace the path from i upstream to the head end; let r_i be the first forward router encountered. Let r'_i be the first point downstream of r_i on the path. As an example of this terminology, i, r_i, r'_i, $t_{i,1}$ and $t_{i,2}$ are shown in one case in Figure 5.1. Note,

*Bridgers are amplifiers which feed into feeder cable from which customer taps and drop lines emenate.

in particular, that even if point e is a local router, it is not labelled $t_{i,e}$ and even if point d is a forward router, it is not labelled r_i.

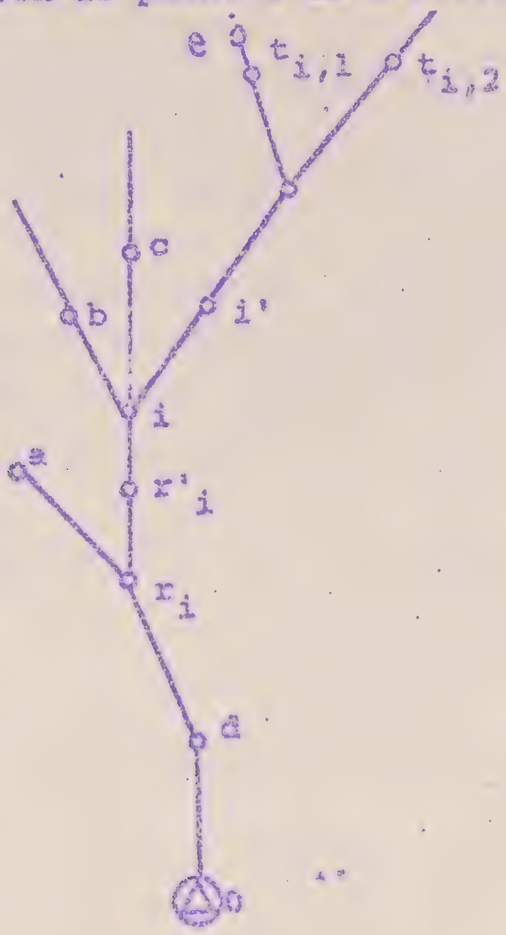


Figure B.1

We next introduce some general terminology to be able to describe sets of points downstream from specified points along possibly specified paths. Let D(a,a') be all points in N downstream of a on the paths containing a'. Let D(a) be all points in N downstream from a. For sets of points A and B chosen from N, I(A x B) is the number of inquiries directed from terminals fed from bridgers in A to terminals fed by bridgers in B and R(A x B) the number of responses directed from points in A to points in B. The total number of messages is M(A,B) = I(A,B) + R(A,B). Thus, for example, M(N x D(i,i')) is the number of messages directed from any point to any point upstream of i on the route containing the router or terminating bridger i'. Finally, we assume:

N_0 = [0], where the head end is point 0,

N_1 = N-N_0, that is all points except the head end,

N_t = N, that is all points.

Then for y=0 the expressions below give the usage of link [i,i'] due to central traffic.

For y=1 the expressions below give the usage of link [i,i'] due to local traffic.

For y=t the expressions use the total traffic to and from all nodes N, including central traffic and local traffic.

We first give the expressions in the case in which there are no converters. Then all forward messages are at ω_f and all reverse messages are at ω_r.

Regardless of the routing at i the traffic in the reverse link [i,i'] is given by:

$$M(D(i,i') \times N_y) - \sum_{\ell} M(D(t_{i,\ell}) \times D(t'_{i,\ell})) \qquad \text{(B.1a)}$$

The expression $M(D(t_{i,\ell}) \times D(t'_{i,\ell}))$ is the local traffic which is prevented by the router at $t_{i,\ell}$ from appearing upstream of $t_{i,\ell}$. The summation gives all such traffic from all local routers downstream of points along the path containing the link [i,i']. Since this summation appears repeatedly, we introduce an abbreviation for it, L(i,i') to indicate the local traffic downstream from i and i' which does not reach i. Rewriting (1a) we then have,

$$M(D(i,i') \times N_y) - L(i,i'). \qquad \text{(B.1b)}$$

The traffic in the forward link [i,i'] is given for the three cases by the following expressions:

A. Local Routing at i:

$$M(N_y \times D(r_i, r'_i)) - L(i,i') \qquad \text{(B2)}$$

B. Forward Routing at i:

$$M(N_y \times D(i,i')) - L(i,i') \qquad (B3)$$

C. No Routing at i:

$$M(N_y \times D(r_i, r'_i)) - L(i,i') \qquad (B4)$$

As an example of the use of the notation, for y=0 the expression in (1) indicates that the traffic in the reverse link is given by the inquiries from the terminals downstream from i on the routes contains i' directed to the head end plus the responses directed from the same terminals to the head end less the inquiries and responses directed from terminals downstream of $t_{i,\ell}$ to other terminals downstream ot $t_{i,\ell}$.

We now consider the addition of converters. For links which are downstream of converters, all messages are at ω_x and ω_z and the above formulae are unchanged. For the remaining links messages are carried at ω_r, ω'_r, ω_f and ω'_f and the formulae must be modified.

The expressions below give the traffic in link [i,i'] at the frequencies ω'_r and ω'_f. We introduce a terminology for concentrators similar to that used for local routers. Trace any path from i in the downstream direction so the path contains i'. The first concentrator is called $c_{i,\ell}$ if it is located at point ℓ .

Regardless of the routing at i', the traffic in the reverse link [i,i'] at ω'_r is:

$$\sum_a M(D(c_{i,a}) \times N_y) - \Big[\sum_{\substack{t_{i,b} \text{ is downstream of} \\ \text{a concentrator } c_{i,c}}} M(D(t_{i,b})) \times D(t_{i,b})) \cdot \sum_{\substack{c_{i,a} \text{ is downstream} \\ \text{of } t_{i,b}}} M(D(c_{i,a})) \times D(t_{i,b})) \Big] \qquad (B.5a)$$

The terms in brackets give the local traffic which does not reach the link [i,i'] because of a local router at $t_{i,b}$. The expression is more complicated than before, because the concentrator may be in two different positions with respect to the router. The first summation in brackets corresponds to the situation in Figure 5.2(a) where the router is downstream of the concentrator. The second summation corresponds to the situation in Figure 5.2(b) where the concentrator is downstream of the router.

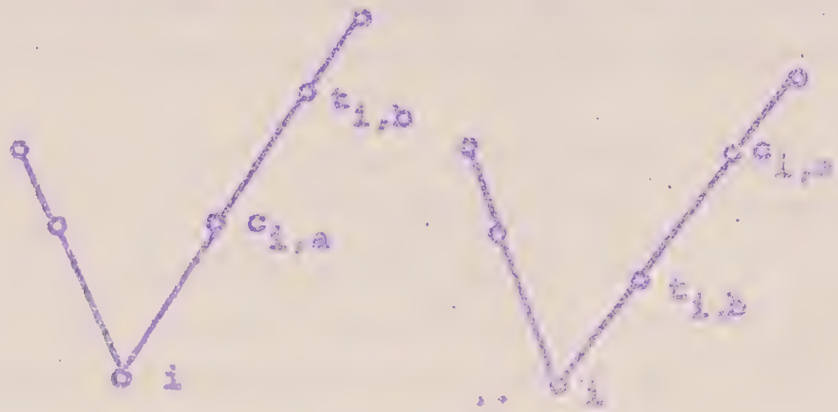


Figure B2

Since the term in brackets appears repeatedly, we introduce an abbreviation for it, $L_c(i,i')$ the local traffic with concentrators. Rewriting (5a) we have,

$$\sum_a M(D(c_{i,a}) \times N_y)) - L_c(i,i') \qquad \text{(B.5b)}$$

The traffic at t_f in the forward link [i,i'] is given by the expressions below. In these expressions A∩B denotes the points common to A and B.

A. Local Routing at i:

$$\sum_a M(N_y \times D(c_{i,a}) \cap D(x,x'_i)) - L_c(i,i') \qquad \text{(B.6)}$$

B. Forward Routing at i:

$$\sum_{a} M(N_y \times D(c_{i,a}) \cap D(i,i')) - L_c(i,i') \qquad (B.7)$$

C. No Routing at i:

$$\sum_{a} M(N_y \times D(c_{i,a}) \cap D(r,r'_i)) - L_c(i,i') \qquad (B.8)$$

To obtain the traffic in these lines at ω_r and ω_f subtract the expressions in (B.6) - (B.8) from the corresponding expressions in (B.2) - (B.4).

The effect of multiplexing at converters is to increase the data rate at ω'_r and ω'_f. The effect of concentration at the converters is to isolate the links feeding upstream into the concentrator; that is each line operates separately at the low data rate, and their combined traffic is handled only at the higher data rate.

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